II/IV B.Tech (Regular/Supplementary) DEGREE EXAMINATION

Electronics and Instrumentation Engineering

Signals & Systems (14EI406/EI226)

Time: Three Hours **Max.Marks: 60 Marks**

Scheme of instructions and solutions

1. Answer all questions (1X12=12 Marks)

a) List out the properties of a power signal.

Ans. **The energy of the power signal is infinity** **1M**
Generally the power signals are periodic in natue.

b) Find the even and odd component of a following signal:x (n) =$e^{-(n/4)}$.

Ans. **x_e(n)= (x(n) +x(-n))/2 = cosh(n/4)** **1M**
x_e(n)= (x(n) +x(-n))/2 = -sinh(n/4)

c) Write down the conditions which are required to be satisfied for the signal to be Fourier transformable.

Ans. **The signal is absolutely integrable over the infinite interval.** **1M**
The signal has finite maxima and minima.
The signal has finite number of discontinuities in every finite time interval and the discontinuities must finite.

d) Check whether the following discrete time system is time invariant (or) not?
y(n)=x(-n).

Ans. **The signal is time invariaent.** **1M**

e) What do you mean by a distortion less transmission through a system.

Ans. **The signal may be amplified or attenuated but the shape of the signal remains same at the output from the input.** **1M**
The signal at the out put may introduce some phase
$y(t) = kx(t-t_0)$

f) Consider a pulse function which is given by ,$f(t) = \begin{cases} \cos \pi t/T & , -T/2 < t < T/2 \\ 0 & , otherwise \end{cases}$

Find Energy spectral density.

Ans. $\frac{1}{2}\left[\delta(\omega-\pi/T)+\delta(\omega+\pi/T)\right]$ $\frac{T\sin(\omega T/2)}{\omega T/2}$ **1M**

$\frac{T^2}{4}\left[\sin^2(\omega T/2-\pi/2)+\sin^2(\omega T/2+\pi/2)\right]$

g) The noise figure of an amplifier is 0.2 db .find the equivalent temperature T_e .

Ans. **$T_e = T_o (F-1) = 14^0K$** **1M**
$T_0 = 27^0C = 273+27 = 300^0K$, 10log(f) = 0.2 => f = $10^{0.02}$

h) An RF amplifier uses an active device with 200Ω equivalent noise resistance and a 300Ω input resistance. The frequency range of the amplifier is from 10 to 12 MHz. Assuming the temperature to be 27^0C, find the noise voltage at the input of the amplifier.

Ans **Vn = sqrt(4KTRΔf) = sqrt(4*1.38x10^{-23}x(200+300)x300x2x10^6)=4.07μV** **1M**

i) Define Thermal noise.

Ans. **Thermal noise, Johnson noise) is the electronic noise generated by the thermal agitation of the charge carriers (usually the electrons) inside an electrical conductor at equilibrium, which happens regardless of any applied voltage.** **1M**

j) Give the properties of conditional distribution function.

Ans. ***$F_X(-\infty|B) = 0$*** ***$F_X(\infty|B) = 1$*** **1M**
$0 \le F_X(-x|B) \le 1$ ***$F_X(-x_1|B) < F_X(-x_2|B)$ then $x_1 < x_2$ any two***

k) If the probability density function of X is given by f(x)=x^2 ,0≤ x ≤ 1.find E(X).

Ans. **1M**

$m_1 = \int_{-\infty}^{\infty} x f_X(x)dx$ =1/4

l) Give the statement for Bayer's theorem.

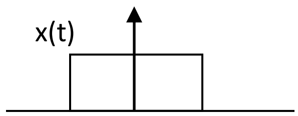
Ans. **Statement: If *B_1,B_2,……….B_n* be a set of exhaustive and mutually exclusive events and A is another event associated with (or caused by) *B_i*, then** **1M**

$$P(A) = \sum_{i=1}^{n} P(B_i).(\frac{A}{B_i})$$

2.a. **Determine whether the following signals are energy signals or power signals and calculate their energy or power.** **6M**
x(t)= rect (t/T$_0$) (ii) x(n)=$(1/3)^n$u(n)

Ans (i) x(t) = rect(t/T_0) **3(M)**

x(t) = 1 $-T_0/2 < t < T_0/2$
0   other wise

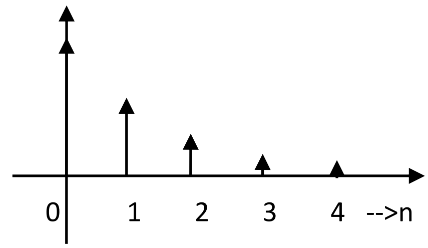


E = $\int_{-\infty}^{\infty} |x(t)|^2 dt$ is finite then it is known as energy signal

$\int_{-T_0/2}^{T_0/2} dt = t\Big|_{-T0/2}^{T0/2} = T_0$ the integral is fine and hence it is energy signal

And the energy of the signal is T_0 jouls

(ii) x(t) = $(1/3)^n$u(n) **3(M)**



E= $\sum_{n=0}^{\infty} x(n) = 1 + \frac{1}{3} + \frac{1}{3^2} + ... + \frac{1}{3^r} + ...\infty$

The summation is finite and hence the signal is energy signal and the energy is

E = $\frac{1}{1-\frac{1}{3}} = \frac{3}{2}$ jouls

2.b. **State and prove the following properties of Fourier transforms.** **6M**
(i)Convolution property in time domain (ii)Differentiation property

Ans. (i) convolution property of F.T **(3M)**

$$g_1(t) \Leftrightarrow G_1(\omega)$$
$$g_2(t) \Leftrightarrow G_2(\omega)$$

Then the fourier trans form of the convolutionof g_1(t) and g_2(t)

F { g_1 (t) ☆ $g_2(t)$ }=

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_1(\tau) g_2(t-\tau) d\tau e^{-j\omega t} dt$$

$$\int_{-\infty}^{\infty} g_1(\tau) \left[\int_{-\infty}^{\infty} g_2(t-\tau) e^{-j\omega t} dt \right] d\tau$$

$$\int_{-\infty}^{\infty} g_1(\tau) e^{-j\omega\tau} \left[\int_{-\infty}^{\infty} g_2(t-\tau) e^{-j\omega(t-\tau)} dt \right] d\tau$$

Assume t-τ = p => dt = dp and the expression is

$$\int_{-\infty}^{\infty} g_1(\tau) e^{-j\omega\tau} \left[\int_{-\infty}^{\infty} g_2(p) e^{-j\omega p} dp \right] d\tau$$

$$\int_{-\infty}^{\infty} g_1(\tau) e^{-j\omega\tau} d\tau \int_{-\infty}^{\infty} g_2(p) e^{-j\omega p} dp$$

$G_1(\omega)G_2(\omega)$

F { g_1 (t) ☆ $g_2(t)$ }= $G_1(\omega)G_2(\omega)$

(ii) Differentitation property of F.T **(3M)**

$$g(t) \Leftrightarrow G(\omega)$$

$$\frac{d}{dt} g(t) \Leftrightarrow j\omega G(\omega)$$

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{j\omega t} d\omega$$

Differentiating on both sides

$$\frac{d}{dt} g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) j\omega e^{j\omega t} d\omega$$

$$\frac{d}{dt} g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [j\omega G(\omega)] e^{j\omega t} d\omega$$

From this we can say that

$$\frac{d}{dt} g(t) \Leftrightarrow j\omega G(\omega)$$

Differentiation in time domain

3.a. **Derive the relationship between trigonometric and exponential Fourier series.**

Ans Relatoin between trigonametric F.s. and Exponential F.S.

Trigonometric Fourier Series for given signal g(t) where g(t) is periodic with period T Is given by

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

$$a_0 + a_1 \cos(\omega_0 t) + a_2 \cos(2\omega_0 t) + ... + a_r \cos(r\omega_0 t) + ... + \infty$$

$$...... + b_1 \sin(\omega_0 t) + b_2 \sin(2\omega_0 t) + ... + b_r \sin(r\omega_0 t) + ... + \infty$$

Where $\cos(n\omega_0 t) = \dfrac{e^{jn\omega_0 t} + e^{-n\omega_0 t}}{2}$ and $\sin(n\omega_0 t) = \dfrac{e^{jn\omega_0 t} - e^{-n\omega_0 t}}{2j}$ by substituting them in place of cos and sin in the above expression

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n\left(\frac{e^{jn\omega_0 t} + e^{-n\omega_0 t}}{2}\right) + b_n\left(\frac{e^{jn\omega_0 t} - e^{-n\omega_0 t}}{2j}\right)$$

$$= a_0 + \sum_{n=1}^{\infty} \frac{a_n}{2} e^{jn\omega_0 t} + \frac{a_n}{2} e^{-jn\omega_0 t} + \frac{b_n}{2j} e^{jn\omega_0 t} - \frac{b_n}{2j} e^{-jn\omega_0 t}$$

$$= a_0 + \sum_{n=1}^{\infty} \left(\frac{a_n}{2} + \frac{b_n}{2j}\right) e^{jn\omega_0 t} + \left(\frac{a_n}{2} - \frac{b_n}{2j}\right) e^{-jn\omega_0 t}$$

$$= a_0 + \sum_{n=1}^{\infty} \left(\frac{a_n - jb_n}{2}\right) e^{jn\omega_0 t} + \left(\frac{a_n + jb_n}{2}\right) e^{-jn\omega_0 t}$$

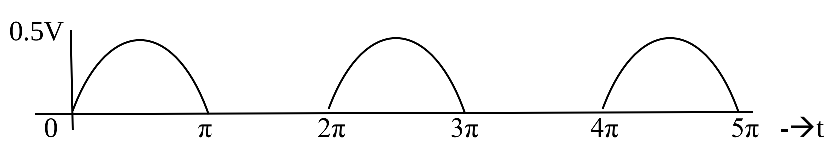
On combining these two we can write exponenetial F.S.

g(t) = $= \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$

wehre $C_n = \left(\dfrac{a_n - jb_n}{2}\right)$

$C_{-n} = \left(\dfrac{a_n + jb_n}{2}\right)$ or viceversa relation.

3.b. Find out the exponential Fourier series for a half wave rectified signal, having the amplitude 0.5 and its period is 2π .Also plot the magnitude & spectrum for the same signal.



Ans. $g(t) = \begin{cases} 0.5\sin(t) & 0 \le t \le \pi \\ 0 & \pi \le t \le 2\pi \end{cases}$ the function is periodic with period 2π

The fundamental frequency $\omega_0 = 2\pi/2\pi = 1$

The exponential F.S. representation of the signal is

$$g(t) = \sum_{n=-\infty}^{n=\infty} C_n\, e^{jn\omega_0 t}$$

$$C_n = \frac{1}{2\pi}\int_0^{\pi} g(t)e^{-jnt}dt = \frac{1}{2\pi}\int_0^{\pi} 0.5\sin(t)e^{-jnt}dt = \frac{1}{2\pi}\int_0^{\pi}\left[\frac{e^{jt} - e^{-jt}}{2j}\right]e^{-jnt}dt$$

$$\frac{1}{4j\pi}\int_0^{\pi} e^{jt(1-n)} - e^{-jt(1+n)}dt = \frac{0.5}{4j\pi}\left\{\frac{e^{jt(1-n)}}{j(1-n)} - \frac{e^{-jt(1+n)}}{-j(1+n)}\right\} \ where\ t\ is\ from\ 0\ to\ \pi$$

On substituting the value of t

$$\frac{0.5}{4j\pi}\left\{\frac{e^{j(1-n)\pi}}{j(1-n)}-\frac{e^{-jt(1+n)\pi}}{-j(1+n)}-\frac{1}{j(1-n)}-\frac{1}{-j(1+n)}\right\}\ for\ n\ is\ even\ that\ is\ n=2,4,6\ etc$$

$$\frac{0.5}{4j\pi}\left\{\frac{-1}{j(1-n)}-\frac{1}{j(1+n)}-\frac{1}{j(1-n)}-\frac{1}{j(1+n)}\right\}$$

$$\frac{1}{4\pi}\left\{\frac{1}{(1-n)}+\frac{1}{(1+n)}\right\}$$

$\frac{1}{2\pi}\left\{\frac{1}{(1-n^2)}\right\}$ n is even

$\frac{1}{2\pi}\left\{\frac{1}{(1-n^2)}\right\}$ n is even

Then expression for the exponential F.S. $=\sum_{n=-\infty}^{n=\infty}\frac{1}{2\pi}\frac{1}{1-n^2}\ e^{jnt}$ for all n is even

$$C_1=\frac{1}{2\pi}\int_0^{\pi}g(t)e^{-jt}dt=\frac{1}{2\pi}\int_0^{\pi}0.5\sin(t)e^{-jt}dt=\frac{0.5}{2\pi}\int_0^{\pi}\left[\frac{e^{jt}-e^{-jt}}{2j}\right]e^{-jt}dt$$

$$C_1=\frac{0.5}{2\pi}\int_0^{\pi}\left[\frac{e^{jt}-e^{-jt}}{2j}\right]e^{-jt}dt=\frac{0.5}{2\pi}\int_0^{\pi}\left[\frac{1-e^{-j2t}}{2j}\right]dt=\frac{0.5}{2\pi}\left[\left[\frac{t}{2j}\right]-\left[\frac{e^{-2t}}{-4j}\right]\right]_0^{\pi}=\frac{1}{8j}$$

The exponential Fourier series $=\left[1+\frac{\mathbf{1}}{\mathbf{8j}}++\sum_{n=-\infty}^{n=\infty}\frac{1}{2\pi}\frac{1}{1-n^2}\ where\ n\neq 0\ \&\ \pm 1\right]$

4.a. **Determine whether the following discrete time systems are linear or non linear .**
i) y(n)=x(n^2+1) (ii) y(n)=x(n)+5

A system is said to be linear then the system should follow the property of superposition and homogenioity.

Homoheniety property means

The system which produces output y(t) for an input of x(t) then it must produce an output ***ay(t)*** for an input of ***ax(t).***

Superposition property means

The system which produces output $y_1(t)$ for an input of $x_1(t)$ and produces $y_2(t)$ for an arbitrary input of $x_2(t)$ then the sytem must produce and outputo $ay_1(t)+by_2(t)$ for an input of $ax_1(t)+bx_2(t)$ ***where a and b are constants.***

$$\mathbf{T[ax_1(t)+bx_2(t)] = aT[x_1(t)]+bT[x_2(t)]}$$

(i) $y(n) = x(n^2+1)$

$y(n) = T[x(n)] = x(n^2+1)$

$x_1(n)$ produces $y_1(n)$

$y_1(n) = T[x_1(n)] = x_1(n^2+1)$

$x_2(n)$ produces $y_2(n)$

$y_2(n) = T[x_2(n)] = x_2(n^2+1)$

$ay_1(n)+b\,y_2(n) = a\,x_1(n^2+1) + bx_2(n^2+1)$

$y_3(n) = T[ax_1(n)+bx_2(n)] = a\,x_1(n^2+1) + bx_2(n^2+1) = ay_1(n)+by_2(n)$

the weighted sum of the outputs is the sum of the inputs hence the system is linear

ii) the same thing is repeated for the second system

y(n) = x(n)+5

the system is linear system

4.b. **Derive the relationship between rise time and bandwidth.**

Relation ship between rise time and band width

The transfer function of the ideal low pass filter is given by

$$H(\omega) = |H(\omega)| e^{-j\omega t_d}$$

$$|H(\omega)| = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$

$$H(\omega) = \begin{cases} e^{-j\omega t_d} & -\omega_c \le \omega \le \omega_c \\ 0 & \omega \ge \omega_c \end{cases}$$

The impulse responce of the lowpass filter

$$h(t) = F^{-1}[H\omega)] = \frac{1}{2\pi} \int_{-\omega c}^{\omega c} e^{-j\omega t_d} d\omega$$

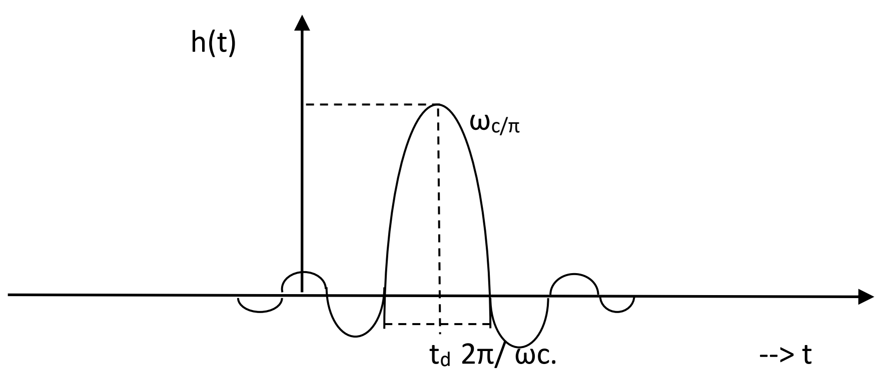
$$h(t) = F^{-1}[H\omega)] = \frac{1}{2\pi} \int_{-\omega c}^{\omega c} e^{-j\omega t_d} e^{-j\omega t} d\omega$$

$$h(t) = \frac{1}{2\pi} \int_{-\omega c}^{\omega c} e^{j\omega(t-td)} d\omega = \frac{1}{2\pi} \left[\frac{e^{j\omega(t-t_d)}}{j(t-t_d)} \right]$$

$$\frac{\omega_c}{\pi} \left[\frac{\sin(\omega_c (t-t_d))}{(\omega_c (t-t_d))} \right]$$

The impulse response has the peak value of (ω_c/t_d) at t = t_d and is proportional to the cutoff frequency.

The width of the main lobe is $2\pi/\omega_c$.



impulse response of ideal low pass filter

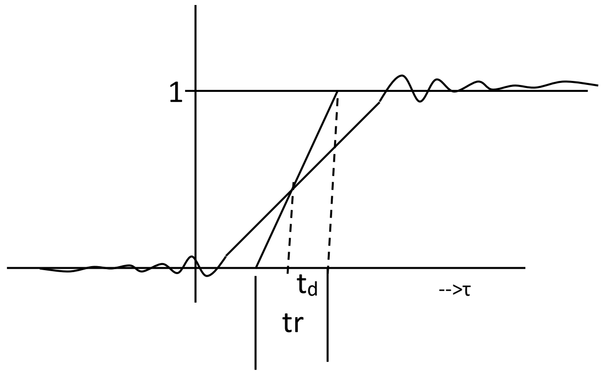
as ω_c --> ∞ the LPF becomes all pass filter

If the impuls eresponce is known then step response can be obtained by convolution

$$y(t) = h(t) \cdot \oplus u(t) = \int_{-\infty}^{t} h(\tau) d\tau$$

$$y(t) = \int_{-\infty}^{t} \frac{\omega_c}{\pi} \frac{\sin(\omega_c(\tau - t_d))}{\omega_c(\tau - t_d)} d\tau$$

$$y(t) = \frac{1}{\pi} [Si(x)]_{-\infty}^{\omega_c(\tau - t_d)}$$



the function y(t) approaches to delayed unit step u(t-t_d).
the rise time tr is defeined as the time required for the response to reach from 0% to 100% of the final value.
Then draw the tangent at t = td with the line y(t = 0 and y(t) = 1

$$\left.\frac{dy(t)}{dt}\right|_{t=td} = \frac{1}{t_r} = \frac{\omega_c}{\pi} \frac{\sin(\omega_c(t - t_d))}{\omega_c(t - t_d)} = \frac{\omega_c}{\pi}$$

$$t_r = \frac{\pi}{\omega_c}$$

Band width x rise time = constant and they are inversely proportional to each other .

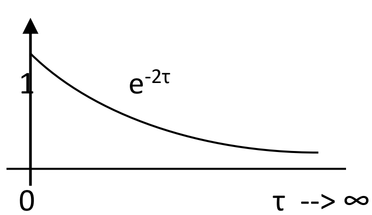
5.a. The impulse response of the circuit is given as h(t)=e^{-2t}u(t).This circuit is excited by an input of x(t)=u(t) .Determine the output of the system.

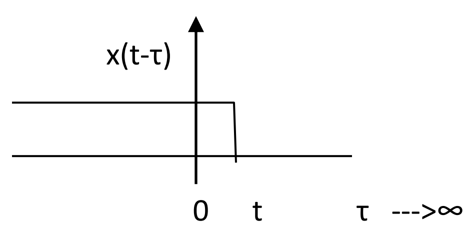
Ans. The impulse response of the circuit is
$h(t)=e^{-2t} u(t)$ --> (1)
The input excitation is x(t) = u(t) -->(2)
The output of the system is y(t) = x(☆) h(t) --->(3)

$$y(t) = \int_{-\infty}^{\infty} h(\tau) x(t - \tau) d\tau$$

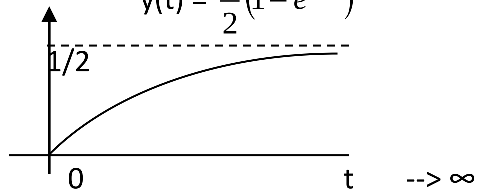
$h(\tau) = e^{-2\tau} \quad 0 \le \tau \le \infty$

$x(t-\tau) = 1 \quad t-\tau \ge 0 \text{ or } \tau \le t$
$\quad\quad 0 \quad$ other wise





$$= \int_{-\infty}^{t} h(\tau) x(t - \tau) d\tau \quad = \int_{0}^{t} e^{-2\tau} d\tau = \left.\frac{e^{-2\tau}}{-2}\right|_{0}^{2} = \frac{1}{2}\left(1 - e^{-2t}\right)$$



5.b. Explain how auto correlation and energy of the signal are related?

The autocorrelation functionof a signal g(t) is given by

$R_g(\tau) = \int_{-\infty}^{\infty} g(t)g(t-\tau)dt$ = g(t) convolution with g(-t) = g(t)☆ g(-t)

g(t) has the fourier trnaform G(ω)

g(-t) has the fourier trnasform G(-ω)

g(t)☆ g(-t) has the F.T as G(ω)G(-ω) = $|G(\omega)|^2$

the energy of the signal is given Reyleigh's energy theorm as

the autocorrelation function is given by

$R_g(\tau) = \int_{-\infty}^{\infty} |G(\omega)|^2 e^{j\omega\tau} d\omega = \int_{-\infty}^{\infty} g(t)g(t-\tau)dt$ at τ=0 then

$= \int_{-\infty}^{\infty} |G(\omega)|^2 d\omega = \int_{-\infty}^{\infty} g(t)^2 dt$ =E

The autocorrelation fuction at τ=0 i.e. $R_g(0)$ = E

6.a. A white noise signal of zero mean and Power spectral density *η*/2 is applied to an ideal Low pass filter whose bandwidth is B.find the autocorrelation of the output noise signal.

Th4e while noise power spectra density is given as η/2 is white noise where the spectrum is constant over the entire spectrum

It is passed through an ideal low pass filter whiose bandwidth is given by B and bothe of them is shown below

Noise spectrum

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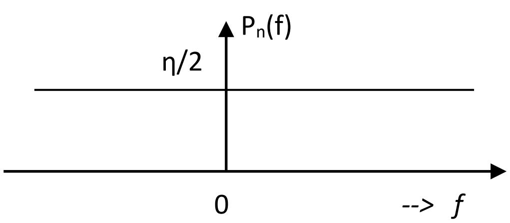
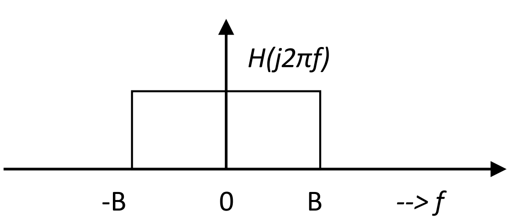




$$P_n(f) = \begin{cases} \frac{\eta}{2} & -B \le f \le B \\ 0 & other \quad wise \end{cases}$$

The noise power density spectrum and autocorrelation function forms fourier transform pair.

Ther inverse Fourier trnansform is given by

R(τ) = $\int_{-B}^{B} \frac{\eta}{2} e^{j2\pi f\tau} df$

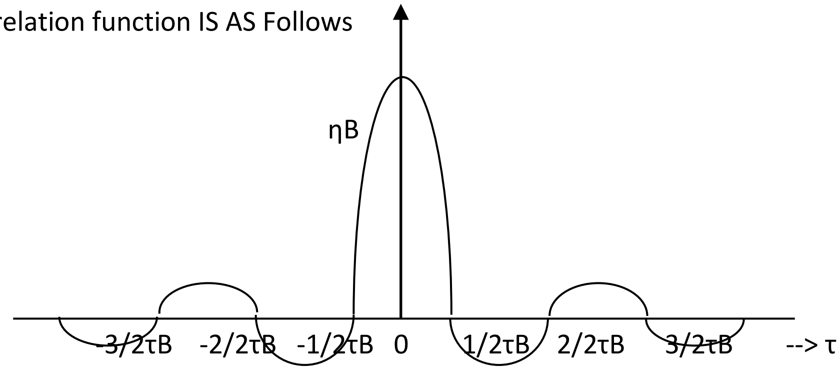
$$= \frac{\eta}{2} \frac{e^{j2\pi f\tau}}{j2\pi\tau}\bigg|_{-B}^{B}$$

$$= \frac{\eta}{j4\pi\tau}\left(e^{j2\pi\tau B} - e^{-j2\pi\tau B}\right)$$

$$= \frac{\eta}{j4\pi\tau} 2j\sin(2\pi\tau B)$$

$$= \eta B \frac{\sin(2\pi\tau B)}{2\pi\tau B}$$

the autocorelation function IS AS Follows



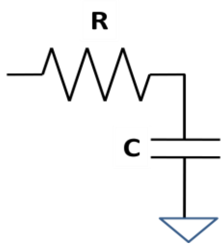
6.b. Calculate the equivalent noise bandwidth of an RC low pass filter. How it is related to its 3db bandwidth.

. Equavalent noise bandwidth of an Rc low pass filter

The Rc low pass filter is shown below

The transfer functio of RC low pass filter is

H(f) = $\dfrac{1}{1+j2\pi fRC}$



Equivalen noise band width

$$= \frac{1}{\|H(0)\|^2}\int_{-\infty}^{\infty}|H(j2\pi f)|^2\,df$$

$$= \int_0^{\infty}\left|\frac{1}{1+j2\pi RfC}\right|^2 df$$

$$= \int_0^{\infty}\frac{1}{1+4\pi^2R^2f^2C^2}\,df$$

$$= \frac{1}{4\pi^2R^2C^2}\int_0^{\infty}\frac{1}{\left(\frac{1}{2\pi RC}\right)^2+f^2}\,df$$

$$= \frac{1}{4\pi^2R^2C^2\,\frac{1}{2\pi RC}}\tan^{-1}2\pi RCf\Big|_0^{\infty}$$

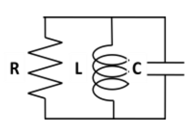
$$= \frac{1}{2\pi RC}\left[\frac{\pi}{2}-0\right] = \frac{1}{4RC}$$

The 3dB band width is double that of the equavalent bandwidth.

7.a. Find the PSD of the thermal noise voltage across the terminals 1 and 2 for the following circuit. Choose R=1MΩ,L=1mH,C=1µf.

Ans. The power spectral density function of the thermal noise voltage across the terminals 1 &2 is

R = 1MΩ , L = 1mH C=



1µF

Impedance of the circuit is given by

$$\frac{1}{Z} = \frac{1}{R}+\frac{1}{j\omega L}+j\omega c$$

$$= \frac{j\omega L+R-\omega^2LCR}{j\omega RL} = \frac{(R-\omega^2LCR)+j\omega L}{j\omega RL}$$

Z $= \dfrac{j\omega LR}{(R-\omega^2LCR)+j\omega L}$ the real and imaginary parts of the impedance is

$Z = Z_{Re} + jZ_{Im} = \dfrac{j\omega LR}{(R-\omega^2LCR)+j\omega L}$

$$= \frac{j\omega LR}{(R-\omega^2 LCR)+j\omega L} \times \frac{(R-\omega^2 LCR)-j\omega L}{(R-\omega^2 LCR)-j\omega L}$$

$$= \frac{j\omega LR(R-\omega^2 LCR)+\omega^2 L^2 R}{(R-\omega^2 LCR)^2+\omega^2 L^2}$$

$$R_{12} = \frac{\omega^2 L^2 R}{(R-\omega^2 LCR)^2+\omega^2 L^2}$$

Then the power density spectrum is given by

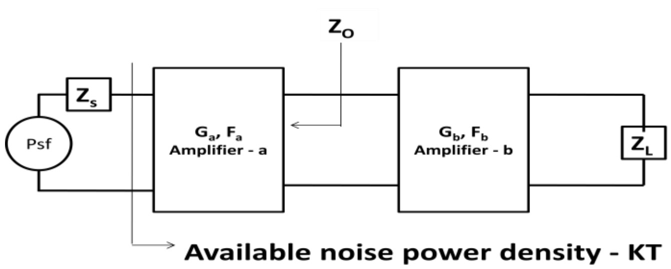
$$PSD = 4KTR_{12} = 4KT\frac{\omega^2 L^2 R}{(R-\omega^2 LCR)^2+\omega^2 L^2}$$

$$= 4KT\frac{\omega^2 10^{-6} 10^6}{(10^6-\omega^2 10^{-3} 10^{-6} 10^6)^2+\omega^2 10^{-6}} = 4KT\frac{10^6\omega^2}{10^6(10^6-\omega^2 10^{-3})^2+\omega^2}$$

$$= 4KT\frac{10^6\omega^2}{(10^9-\omega^2)^2+\omega^2} = \frac{10^6}{(10^9\omega^{-1}-\omega)^2+1}$$

7.b. Derive an expression for overall noise figure of cascades stage amplifiers.

Ans. Noise figure of cascaded stage amplifier



$(P_{no})_{av} = (F_b-1)g_b KT$

$(P_{nto})_{av} = F_{ab}\, g_{ab} KT$

$g_{ab} = g_a.g_b$

$(P_{nto})_{av} = F_{ab}\, g_a.g_b\, KT$

Noise power available at the output of the cascaded stage amplifier is

= noise power due to two srage amplifer + noise power due to thesecond stage of the amplifier

$(P_{nto})_{av} = F_{ab}\, g_a.g_b\, KT = (F_b-1)g_b KT + F_a\, g_a.g_b\, KT$

$F_{ab}\, g_a.g_b\, KT = (F_b-1)g_b KT + F_a\, g_a.g_b\, KT$

$$F_{ab} g_a g_b KT = (F_b - 1) g_b KT + F_a g_a g_b KT$$

$$F_{ab} = \frac{(F_b - 1) g_b KT}{g_a g_b KT} + \frac{F_a g_a g_b KT}{g_a g_b KT}$$

$$F_{ab} = F_a + \frac{(F_b - 1)}{g_a}$$

For n stage amplifier it is given by

$$F = F_1 + \frac{(F_2 - 1)}{g_1} + \frac{(F_3 - 1)}{g_1 g_2} + \frac{(F_4 - 1)}{g_1 g_2 g_3} + \dots\dots\dots\dots\dots\dots$$

8.a. Define probability density function and write its properties

Ans. Probability density function and properties

The probability density function is denoted by $f_X(x)$ and is defined from the distribution function as

$$f_X(x) = \frac{dF_X(x)}{dx}$$

Then the function Is called the density function of the random variable X and the Properties of

$$f_X(x)$$

the density function

1. $0 \leq f_X(x)$ for all x

2. $$\int_{-\infty}^{\infty} f_X(x)dx = 1$$

3. $$F_X(x) = \int_{-\infty}^{x} f_X(x)dx$$

4. $$P\{x_1 < X \leq x_2\} = \int_{x1}^{x2} f_X(x)dx$$

8.b. **State and prove the Bayer's theorem.** **6M**

Ans. **Total Probability Theorem:**

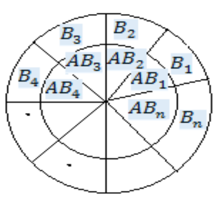
Statement: If $B_1, B_2, \ldots\ldots\ldots B_n$ be a set of exhaustive and mutually exclusive events and A is another event associated with (or caused by) B_i, then

$$P(A) = \sum_{i=1}^{n} P(B_i).(\frac{A}{B_i})$$

(3M)

Proof:

The inner circle represents the event A. A can occur along with (or due to) $B_1, B_2, \ldots\ldots\ldots\ldots B_n$ that are exhaustive and mutually exclusive.



$\therefore AB_1, AB_2, AB_3, AB_4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots AB_n$ are also mutually exclusive

$\therefore A = AB_1 + AB_2 + AB_3 + AB_4 \ldots\ldots + AB_n$ (By Addition Theorem)

$$\therefore P(A) = P(\sum_{i=1}^{n} AB_i)$$

$$= P(\sum_{i=1}^{n} PAB_i)$$

$\therefore P(A) = \sum_{i=1}^{n} P(B_i).P(\frac{A}{B_i})$...(A) (Using conditional probability

$$P(AB) = P(A \cap B) = P(B).P(B/A) = P(A).P(A/B))$$

Bayes' Theorem or Theorem of Probability of causes

Statement: If $B_1, B_2, \ldots\ldots\ldots. B_n$ be a set of exhaustive and mutually exclusive events associated with a random experiment and A is another event associated with (or caused by) B_i, then

$$P(\frac{B_i}{A}) = \frac{P(B_i).P(\frac{A}{B_i})}{\sum_{i=1}^{n} PB_i.P(\frac{A}{B_i})} \ldots\ldots i = 1, 2..n$$

Proof:

We know Conditional Probability is given as:

$$P(AB_i) = P(A \cap B_i) = P(B_i).P(A/B_i) = P(A).P(B_i/A) - - - - - - - - - (1)$$

$$P(\frac{B_i}{A}) = \frac{P(B_i).P(\frac{A}{B_i})}{P(A)} \ldots\ldots (2)$$

Now using Total Probability Theorem we have,

$$P(A) = \sum_{i=1}^{n} PB_i.P(\frac{A}{B_i}) - - - - - -(3)$$

From equation (2) and equation (3)

$$P(\frac{B_i}{A}) = \frac{P(B_i).P(\frac{A}{B_i})}{\sum_{i=1}^{n} PB_i.P(\frac{A}{B_i})} \ldots\ldots$$

(3M)

9.a Let X be a random variable with probability density function **6M**

$$f_x(x) = \begin{cases} 5x^2; & 0 < x < 1 \\ 0 \;; & \text{otherwise} \end{cases}$$

find (i) E[X-4] (ii) E[5X+2]

Ans. Random variable X

Probability density function

$$f_X(x) = \begin{cases} 5x^2 & 0 \le x \le 1 \\ 0 & other\,wise \end{cases}$$

The E[x-4] = $\int_{-\infty}^{\infty} (x-4) f_X(x)dx$ = $\int_0^1 (x-4)5x^2 dx$ $= 5\int_0^1 (x^3 - 4x^2)dx$

$= 5\left[\frac{x^4}{4} - 4\frac{x^3}{3}\right]_0^1$ = $5\left[\frac{1}{4} - \frac{4}{3}\right]$ = $5\left[\frac{3-16}{12}\right]$ $= \frac{-65}{12}$ **(3M)**

The E[5x+2]

m_0 = $\int_{-\infty}^{\infty} f_X(x)dx$ = $\int_0^1 5x^2 dx$ = $5\left[\frac{x^3}{3}\right]_0^1$ = $\frac{5}{3}$

m_1 = $\int_{-\infty}^{\infty} x f_X(x)dx$ = $\int_0^1 x.5x^2 dx$ = $5\left[\frac{x^4}{4}\right]_0^1$ = $\frac{5}{4}$

E[5x+2] = $5m_1$-$2m_0$ = $\frac{25}{4} + \frac{10}{3} = \frac{115}{12}$ **(3M)**

9.b. **A company produces electric relays has three manufacturing plants producing 60, 40and 10 percent respectively of its product. Suppose that the probability that relaymanufactured by these plants is defective are 0.02, 0.03and 0.01 respectively. If a relayis selected at random found to be defective, what is the probability that it is came fromplant 3?** 6M

Ans. There are three manufacturing plants P1,P2,P3

The components produced by each plant in terms of % age are

P1	P2	P3
60%	40%	10%

Probability of producing defective component by each plant is

P1	P2	P3
0.02	0.03	0.01

The probability to select a plant randomly is = 1/3 **(3M)**

Probability to select a defective component from plant 3 is = probability to select plant 3 and the selected one is defective = $\frac{1}{3} * 0.01 * \frac{10}{100} = \frac{1}{3} \times 10^{-3}$ **(3M)**